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MODEL OF RELIABILITY OF PLANETARY GEARS

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Abstract. A model of the reliability of planetary gears is described, in which the epicycle of a stationary sun gear is connected to the input shaft and the carrier is connected to the output shaft. The input and output shafts are coaxial, and it is assumed that these shafts are loaded only with torques *be3* and bending forces. Axial and lateral forces are not considered. Gears of this type are usually used in helicopter main rotor drive systems, as well as in other cases where it is necessary to provide a large transmission ratio. The reliability model is based on a Weibull distribution of the reliability of individual gear elements. The load-bearing capacity of a gear under dynamic loading conditions is the value of torque on the input shaft that can be used to load the gear with a design life of 106 revolutions of the sun gear. Load and durability are related by a power law. The exponential curve in coordinates load - durability and load-bearing capacity under dynamic loading conditions are determined as a function of the load-bearing abilities of the transmission parts.

Keywords: satellite, epicycle, rotation angles, planetary gear, dynamic carrier, tangential direction, loading cycles.

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Designations

$C-$ initial dynamic bearing capacity of the element, N_m; $D-$ initial dynamic bearing capacity of the system, N_m; $e-$ Weibull curve degrees index; $f-$ width of the gear rim, m; $F-$ force, N; $l-$ element durability in 10^6 loading cycles; $L-$ durability in 10^6 revolutions of the sun; $n-$ number of satellites; $N-$ number of gear teeth; $p-$ indicator of the dependence degree between load and durability; $R-$ radius of the initial circle of the gear wheel, m; $S-$ probability of absence of destruction (reliability); T_i- input torque, N_m; $V-$ stressed volume m^3; $z-$ depth to maximum tangential stresses, m; $\sum_\rho$ – sum of curvatures at the engagement pole, $1/m$; $\theta-$ rotation angle, *degrees;* $\tau-$ maximum shear stress, Pa; $\varphi-$ profile angle on the pole line, *degrees;*

Indexes

A – carrier; B – satellite bearings; P – satellite; PR – satellite engaged with epicycles; PR - satellite that engaged with the sun; r – radial direction; R – epicycle; S – sun; t – tangential direction; T – transmission; *10* – corresponds to a 90% probability of no destruction.

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1 Introduction

In recent years, it has been generally accepted that when designing mechanical systems and their elements, the influence of actual factors should be more accurately taken into account, and not limited to considering the case of a constant load and constant mechanical characteristics (Bury, 1974; Haugen, 1980; Smith, 1976). When designing a structure with given properties, it is not economical to carry out calculations with a simple safety factor. An approach based on probabilistic calculation methods is more consistent with reality. In this case, the design is calculated based on the assumption that the load and mechanical characteristics change statistically, which is closer to the actual situation. The statistical (probabilistic) calculation method, the use of which requires not only knowledge of the nominal values of the acting forces and mechanical characteristics, but also their static changes, allows the designer to determine *the reliability or probability of non-destruction* of a mechanical system (Haugen, 1980; Smith, 1976). It is impossible to determine them using only the safety factor.

Feasibility of the use of probabilistic calculation methods is most obvious in the design of aircraft drives, when it is necessary to reconcile the requirements for weight reduction, high power density and high speeds, on the one hand, and reliability, ease of operation, and long turnaround times. timing - on the other. There is currently no acceptable probabilistic method for calculating lightweight planetary gears intended for use in helicopters.

The probabilistic method was used to calculate machine components by Hogen and Smith, and to calculate planetary gears by Rao (Rao, 1979). These methods are based on the assumption that the acting forces and mechanical characteristics of parts obey a Gaussian distribution. Both methods also assumed the existence of an endurance limit, i.e. ultimate stress at which parts and assemblies have infinite durability.

It has been shown by Lundberg and Palmgren (Kerimov, 2002; Aliyev, 2023), as well as Coy, Townsend and Zaretsky (Coy, 1975; Lundberg & Palmgren, 1952; Aliyev & Aliyeva, 2023) that roller bearings and teeth of high-strength steel gears have some durability at any amount of applied force. The static model of the durability and load-bearing capacity of these parts obeys the Weibull distribution law (Kerimov, 2002; Lundberg & Palmgren, 1952; Aliyev & Aliyeva, 2023; 2017; Aliyeva et al., 2021). The reason is that both bearings and gears are subjected to contact fatigue failures.

In the planetary gear calculation method given by Rao (Bury, 1974; Haugen, 1980; Smith, 1976), changes in the operational load are taken into account, and the possibility of a fracture of a contact destruction of a fatigue nature is allowed. However, the calculation does not take into account the durability of the satellite bearings and the durability of the entire system. The parallel forces acting on the satellites are considered to statically increase the reliability of the system. In reality, satellite bearings have a significant impact on the durability of the entire assembly, and fragments from a failed part in a drive operating at high speed may well cause failure of the entire assembly. Thus, in order to create a sufficiently reliable model of a planetary gear, it is necessary to strictly take into account the reliability of a number of elements included in it.

Based on the above provisions, we set the goal of creating a model of the reliability of planetary gears used in helicopter main rotor drives. Therefore, the following kinematic diagram of the planetary gear is considered here: the epicycle is braked, the input shaft is connected to the sun gear, and the satellite carrier is connected to the output shaft. It is assumed that the input and output shafts are coaxial and loaded only with torque. The reliability model of individual gears and bearings follows a Weibull distribution. The durability of the system with a 90% probability of absence of destruction, calculated from the corresponding lives of individual elements, is taken as the reliability of the drive. The initial dynamic load-bearing capacity of the system is considered to be the value of the input torque at which the input shaft connected to the sun gear can make 10^6 revolutions with a 90% probability of no destruction. The change in durability depending on the load at a given reliability is modeled by a power law. When plotted in a logarithmic coordinate system, this curve becomes a straight line. It is believed that this dependence at a given load is not related to the Weibull dependence of reliability on durability (Kerimov, 2002; Aliyev, 2023; Aliyev & Aliyeva, 2017). The exponent depending on the durability on the load and the initial dynamic load-bearing capacity are obtained as functions of the load-bearing abilities of the elements.

2 Kinetics and kinematics

The most common scheme of the transmission in question is presented in Figure 1. The satellites are made of two crowns: the inner crown engages with the sun gear, and the outer one with the epicycle. The outer and inner crowns are made as one piece with a bearing located along the axis. The axles on which the bearings sit are connected to the carrier, which is a low-speed link connected to the output shaft. The number of satellites may vary, but they are all the same. It is assumed that the load transmitted by the satellites is the same.

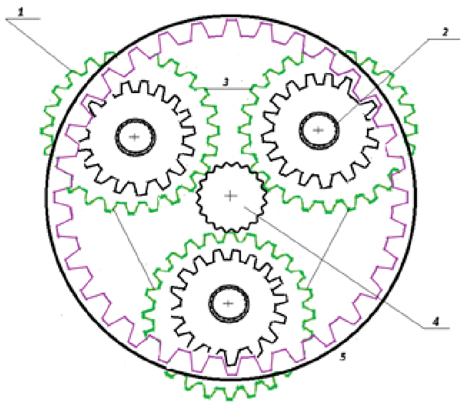

Figure 1: Planetary gear; 1 – double-crown satellite; 2 – satellite bearing; 3 – carrier connected to the output shaft; 4 – sun connected to the input shaft; 5 – inhibited epicycle

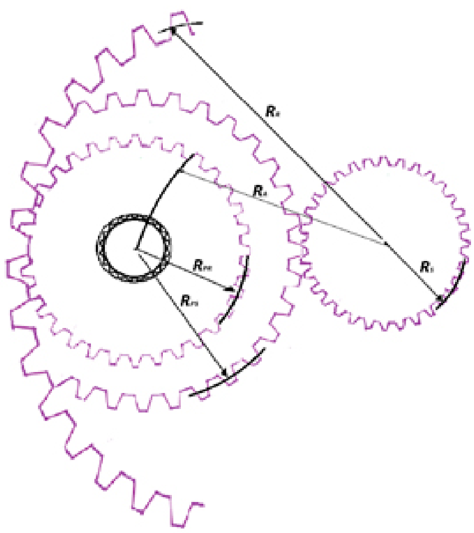

Figure 2: Geometric relationship with planetary gear

One of the satellites engaged with the sun gear and the epicycle and connected to the carrier, with point A being the center of the satellite is shown in figure 2. The radiuses of the carrier and epicycle are related to the radiuses of the sun gear and satellites in accordance with the equations

$$R_A = R_S + R_{RS}, \tag{1}$$

$$R_R = R_A + R_{PR} = R_S + R_{RS} + R_{PR}. \tag{2}$$

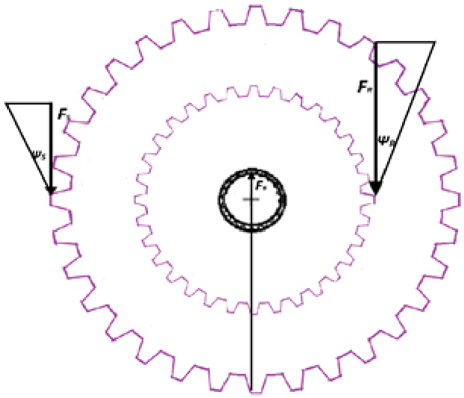

Figure 3: Forces acting on the satellite

Figure 3 shows a diagram of the forces acting on the satellite. The tangential components of the forces, tangent to the initial circles F_S and F_R, expressed through the moment at the input T_i, are determined by the formulas

$$F_S = \frac{T_i}{nR_S}, \tag{3}$$

$$F_R = \left(\frac{R_{PS}}{R_{PR}}\right) F_S = \left(\frac{R_{PS}}{R_{PR}}\right) \left[\frac{T_i}{nR_S}\right]. \tag{4}$$

The sum of the tangential components acts on the bearing

$$F_t = F_S + F_R = \left(\frac{R_{PR} + R_{PS}}{R_{PR}}\right) \left[\frac{T_i}{nR_S}\right]. \tag{5}$$

The total load on the bearing also includes a radial component due to the radial components of the forces on the teeth

$$F_r = F_R tg\varphi_R - F_S tg\varphi_R. \tag{6}$$

Thus, the total load on the bearing is equal to

$$F_B = \sqrt{F_t^2 + F_r^2}. \tag{7}$$

It should be noted that when calculating, the forces acting on the satellite are assumed to lie all the time in the radial plane of the transmission, therefore the vectors of forces acting on the bearings do not change their position. In the case where the satellite is made multi-crown, this condition can be satisfied if the satellite is symmetrical in the axial direction.

It's necessary to carry out a kinematic analysis of the planetary gear in order to determine the relative number of loading cycles that each element undergoes during one revolution of the sun gear. This is necessary for calculating durability limited by fatigue strength. Such kinematic analysis was performed in (Ragimova, 2013). The results are presented in the table, where the angle of rotation of the sun gear is presented. All rotation angles are determined in a fixed coordinate system associated with the inhibited epicycle. Considering the rows of the table, we get for each i - element the expression for the relative angle of rotation using the formula

$$\theta_i = \theta_{i/A} + \theta_A, \tag{8}$$

where A refers to the carrier.

Table 1: The angle of rotation of the planetary gear elements, expressed in terms of the angle of the sun gear rotation

i	θ_i	$= \theta_{i/A}$	$+\theta_A$
Sun	θ_S	$\frac{\theta_S}{1+\frac{R_S}{R_R}\left(\frac{R_{PR}}{R_{PS}}\right)}$	$\frac{\theta_S}{1+\frac{R_R}{R_S}\left(\frac{R_{PS}}{R_{PR}}\right)}$
Satellite	$\frac{\left(1-\frac{R_R}{R_{PR}}\right)\theta_S}{1+\frac{R_R}{R_S}\left(\frac{R_{PS}}{R_{PR}}\right)}$	$\frac{-\theta_S}{\frac{R_{RS}}{R_S}+\frac{R_{PR}}{R_R}}$	$\frac{\theta_S}{1+\frac{R_P}{R_S}\left(\frac{R_{PS}}{R_{PR}}\right)}$
Epicycle	0	$\frac{-\theta_S}{1+\frac{R_R}{R_S}+\left(\frac{R_{PS}}{R_{PR}}\right)}$	$\frac{\theta_S}{1+\frac{R_R}{R_S}\left(\frac{R_{PS}}{R_{PR}}\right)}$
Carrier	$\frac{\theta_S}{1+\frac{R_R}{R_S}\left(\frac{R_{PS}}{R_{PR}}\right)}$	0	$\frac{\theta_S}{1+\frac{R_R}{R_S}\left(\frac{R_{PS}}{R_{PR}}\right)}$

The values of the rotation angles of the corresponding elements given in the table can be used to calculate the number of cycles of their loading based on the number of revolutions of the shaft connected to the sun.

3 Reliability of satellite bearings and their load-bearing capacity

The reliability and load-bearing capacity of a planetary gear depend on the reliability and load-bearing capacity of its parts. These values are quite accurately determined for bearings (Kerimov, 2002; Lundberg & Palmgren, 1952; Rahimova & Mansurova, 2022). The fatigue life model, proposed in 1947 by Lundberg and Palmgren (Kerimov, 2002), is still widely used in calculations up to this day. The reliability of an individual bearing can be expressed through the probability of its non-destruction S with durability determined by the number l revolutions using the following equation:

$$\log\frac{1}{S} \sim \tau^c l^{e_W}\left(\frac{V}{z^h}\right), \tag{9}$$

where τ is the critical value of tangential stresses at a depth z from the surface, and V is the stressed volume. The degree indices are obtained based on the test results of several groups of bearings under the same conditions. Exponent in Weibull relation e_W is a measure of dispersion in the distribution of bearing lives.

The previous formula for the probability of non-destruction reflects the observed influence of stresses, stressed zone and loading cycles on reliability. Increasing the voltage τ reduces reliability. When the stressed zone is located closer to the surface (smaller z), reliability also decreases. Presumably this is explained by the fact that a microcrack begins to develop at the point of maximum stress, located under the surface, and it takes some time for it to reach the surface. Therefore, for any number of loading cycles, the greater probability of a crack emerging, the closer the stressed zone is located to it.

The volume of the stressed zone has also a significant value. Fatigue spalling begins near any even small stress raiser present in the material of the part. The greater the stressed volume, the greater the probability of failure.

For a given load and bearing geometry, the tenth logarithm of the life l_{10}, corresponding to a probability of non-failure of 90%, can be written,

$$\log\frac{1}{S} = \log\frac{1}{0,9}\cdot\left(\frac{l_B}{l_{B10}}\right)^{e_W}. \tag{10}$$

The relationship between the durability of a bearing and the load acting on it, corresponding to a 90% probability of non-destruction, has the form:

$$l_{B10} = \left(\frac{C_B}{F_B}\right)^{p_W}. \tag{11}$$

The initial dynamic load capacity is the load that can be supported by 90% of the bearings during 106 revolutions of the inner ring under specified operating conditions.

To make it easier to bring the durability and load-bearing capacity of all drive parts to the durability and load-bearing capacity of the drive as a whole, let us express each of them through the rotational speed to the torque on the input shaft connected to the sun gear.

From the table we obtain the rotation frequency of the inner ring of the bearing through the rotation frequency of the sun gear

$$\theta_B = \frac{\theta_S}{\frac{R_{PS}}{R_S}+\frac{R_{PR}}{R_R}}. \tag{12}$$

Denoting further durability of parts by the lowercase letter l, expressed through the number of their cycles, and durability with the capital letter L, expressed through the rotation frequency of the input shaft connected to the sun gear, we rewrite equations (10) taking into account (12) in the form

$$\log\frac{1}{S} = \log\frac{1}{0,9}\cdot\left\{\frac{R_S R_R L_B}{\left(R_R R_{PS}+R_S R_{PR}\right) l_{B10}}\right\}^{e_W}. \tag{13}$$

Considering the probability of non-destruction of the satellite bearing to be equal to 90% (i.e. $S_B = 0,9$), and $L_B = L_{B10}$ the revolutions of the sun gear and substituting them into (13), the following is obtained:

$$L_{B10} = \left(\frac{R_R R_{PS}+R_S R_{PR}}{R_R R_S}\right) l_{B10}, \tag{14}$$

that was assumed from consideration of equation (12).

To obtain the dependence of durability on the load for bearings, expressed through the input parameters of the drive, it's necessary to substitute expressions for the load on the bearing, expressed through the moment on the input shaft, determined by equations (3)-(7), into equation (11), and then to substitute all this into equation (14)

$$L_{B10} = \left[\frac{R_R R_{PS}+R_S R_{PR}}{R_R R_S}\right]\times\left\{\frac{n R_S C_B}{T_i\left[\left(\frac{R_{PR}+R_{PS}}{R_{PR}}\right)^2+\left(\frac{R_{PS}}{R_{PR}}tg\varphi_R - tg\varphi_S\right)^2\right]^{\frac{1}{2}}}\right\}^{p_W} \tag{15}$$

The dynamic load-bearing capacity of the satellite bearing is equal to the torque on the input shaft connected with the sun gear, which the sun gear bearings can absorb with a 90% probability of non-destruction during 106 revolutions of the sun gear. Using formula (15), we obtain the dynamic load-bearing capacity of the bearing system equal $T_i = D_B$ to at $L_{B10} = 1,0$. As a result, the formula for calculating the dynamic load-bearing capacity takes the form:

$$D_B = \left(\frac{R_{PS}R_R + R_{PR}R_S}{R_R R_S}\right)^{\frac{1}{p_W}}\times\left\{\frac{n R_S C_B}{\left[\left(\frac{R_{PR}+R_{PS}}{R_{PR}}\right)^2+\left(\frac{R_{PS}}{R_{PR}}tg\varphi_R - tg\varphi_S\right)^2\right]^{\frac{1}{2}}}\right\} \tag{16}$$

The relationship between the durability of a bearing in 106 revolutions of the sun gear with such a torque applied to the sun gear that 90% of the bearings will not fail, takes the form:

$$L_{B10} = \left(\frac{D_B}{T_i}\right)^{p_W} \tag{17}$$

So, the main quantities that characterize the reliability and durability distribution curve of individual bearings and bearings included in the drive system have been defined. Let us finally write down the equation for the probability distribution curve of durability satellite bearings in the form:

$$\log\frac{1}{S_B} = \log\frac{1}{0,9} \cdot \left(\frac{L_B}{L_{B10}}\right)^{e_W} \tag{18}$$

where L_B is the number of millions of revolutions of the sun gear at which the probability of non-destruction of the bearing is equal to S_B.

4 Reliability of the sun gear and its load-bearing capacity

Contact fatigue life and load-bearing capacity of a spur gear were the topic of recently published works (Aliyev, 2023; Coy, 1975; Aliyev & Aliyeva, 2023.). In these works, the above-mentioned Lundberg–Palmgren reliability model is applied to spur gears.

Tests have shown that the fatigue life of gears follows the specified reliability curve, but with a different Weibull dependence than for a bearing.

$$\log\frac{1}{S} = \log\frac{1}{0,9} \cdot \left(\frac{l}{l_{10}}\right)^{e_G}, \tag{19}$$

where S is the probability of non-fracture of one gear tooth, and l is the number of cycles of loading the tooth surface.

The dependence of durability on load for one tooth with a 90% probability of its non-destruction has the form:

$$l_{10} = \left(\frac{C_t}{F}\right)^{P_G}, \tag{20}$$

where F is the transmitted tangential component of the force on the tooth, and C_t- initial dynamic load-bearing capacity of the tooth, which was obtained in the work (Aliyev & Aliyeva, 2023);

$$C_t = \frac{B_1 f}{\sum_\rho}, \tag{21}$$

where f is active crown width, $\sum_\rho$ -the sum of curvatures in the engagement pole, and the constant B_1, measured in pressure units (Pa), is obtained empirically when determining the durability of the spur gear (Aliyev & Aliyeva, 2023). For steel AISI 9310, produced by melting in a vacuum arc furnace and subjected to surface hardening, $B_1 = 135\ MPa$.

In this case, the mentioned basic gear tooth reliability equation is applied to the sun gear. Since there are n satellites, then during the time when the sun gear makes a certain number of revolutions relative to the carrier $L_{S/A}$ each tooth of the sun gear undergoes $nL_{S/A}$ cycles. In the table, the number of loading cycles LS is expressed through the number of revolutions of the sun gear LS , namely

$$l_S = \frac{nL_S}{1 + \frac{R_S R_{PR}}{R_R R_{PS}}}. \tag{22}$$

Probability of non-destruction of the sun gear SS is obtained using the law of factors for a given number of sun gear teeth NS :

$$S_S = S^{N_S}. \tag{23}$$

Using equations (19), (22) and (23), we obtain an expression for the reliability of the sun gear:

$$\log\frac{1}{S} = N_S\log\frac{1}{0,9}\left\{\left(\frac{nR_RR_{PS}}{R_RR_{PS}+R_SR_{PR}}\right)\left(\frac{L_S}{l_{S10}}\right)\right\}^{e_G}, \tag{24}$$

where L_S is the number of millions of revolutions of the sun gear, corresponding to the probability S_S of its non-destruction, l_{S10} is the number of millions of loading cycles that one tooth of the sun gear can withstand with a 90% probability of non-destruction. Equation (24) is completely similar to equation (13) for satellite bearings. Using the same reasoning, we can assume that the 90% probability of non-destruction of the sun gear, expressed in its revolutions, is associated with the 90% probability of non-destruction of one sun gear tooth, expressed in loading cycles in the form:

$$L_{S10} = \left(\frac{1}{N_S}\right)^{1/e_G}\left\{\frac{R_RR_{PS}+R_SR_{PR}}{nR_RR_{PS}}\right\}l_{S10}. \tag{25}$$

As in the case of satellite bearings, in order to obtain a formula for the dynamic load-bearing capacity D_S of the sun gear, expressed through the dynamic load-bearing capacity of an individual sun gear tooth C_S at L_{S10}, equal to 106 cycles, one should substitute the dependence (20) of durability on the load per tooth in equation (25) and express transmitted load through equation (3) for input torque

$$\mathrm{D_S} = \left(\frac{1}{N_S}\right)^{\frac{1}{e_GP_G}}\left[\frac{R_RR_{PS}+R_SR_{PR}}{nR_RR_{PS}}\right]^{\frac{1}{P_G}} nR_RC_S \tag{26}$$

The relationship between the longevity of the sun gear and the moment applied to the shaft connected to it, at which in 90% of cases the sun gear is not destroyed, is obtained in the form:

$$L_{S10} = \left(\frac{D_S}{T_i}\right)^{P_G}. \tag{27}$$

Finally, the probability distribution curve for the reliability of the sun gear can be written as follows:

$$\log\frac{1}{S_S} = \log\frac{1}{0,9}\cdot\left(\frac{L_S}{L_{S10}}\right)^{e_G}, \tag{28}$$

where L_S is the number of millions of revolutions of the sun gear at which the probability of its non-destruction is equal to S_S.

5 Reliability and bearing capacity of the epicycle

The reliability and dynamic load-bearing capacity of the epicycle are calculated in the same way as in the case of the sun gear: equations (19)-(21) are valid for calculating the teeth of the epicycle. Due to the differences in the engagement geometry of the sun gears and the epicycle, the durability and magnitude of the dynamic load-bearing abilities of their teeth differ. With the same modules and the width of the crowns, equal to the width of the sun gear crown, the reliability and initial dynamic load-bearing capacity of the epicycle should be significantly higher due to more uniform contact of the internal teeth of the epicycle and the external teeth of the satellites. The equation that determines the relationship between the number of loading cycles of the epicycle l_R and the number of revolutions of the sun gear L_R, taken from the table, has the form:

$$l_R = \frac{nL_R}{1+\frac{R_RR_{PS}}{R_SR_{PR}}}. \tag{29}$$

The probability of non-destruction of the epicycle $S_R$is obtained directly using the law of factors for a certain number $N_R$of epicycle teeth:

$$S_R = S^{N_R}, \tag{30}$$

where S is the probability of non-destruction of one epicycle tooth. Solving equations (19), (29) and (30) together, we obtain the formula for the reliability of the epicycle:

$$\log \frac{1}{S_R} = N_R \log \frac{1}{0,9} \cdot \left\{ \frac{n R_S R_{PR} L_R}{(R_S R_{PR} + R_R R_{PS})\, l_{R10}} \right\}^{e_G}. \tag{31}$$

The value of 90% durability of the epicycle, expressed in terms of sun gear revolutions, can be associated with the 90% durability of one epicycle tooth, expressed in terms of the number of tooth loading cycles:

$$L_{R10} = \left(\frac{1}{N_R} \right)^{\frac{1}{e_G}} \left\{ \frac{R_S R_{PR} + R_R R_{PS}}{n R_S R_{PR}} \right\} l_{R10}. \tag{32}$$

The initial dynamic load-bearing capacity D_R of the epicycle is equal to the torque on the input shaft connected with the sun gear, at which the probability of non-destruction of the epicycle is 90% for 106 revolutions of the sun gear:

$$D_R = \left(\frac{1}{N_R} \right)^{\frac{1}{e_G P_G}} \left[\frac{R_S R_{PR} + R_R R_{PS}}{n R_S R_{PR}} \right]^{\frac{1}{P_G}} \cdot \left\{ \frac{n R_S R_{PR}}{R_{PS}} C_R \right\}. \tag{33}$$

The formula for the durability of the epicycle, expressed through the magnitude of the torque applied to the shaft connected to the sun gear with a 90% probability of non-destruction of the epicycle, has the form:

$$L_{R10} = \left(\frac{D_R}{T_i} \right)^{P_G}. \tag{34}$$

So, the initial values of the reliability and durability of the epicycle have been determined. The equations of the epicycle reliability probability distribution curve can be written in the following form:

$$\log \frac{1}{S_R} = \log \frac{1}{0,9} \cdot \left(\frac{L_R}{L_{R10}} \right)^{e_G}, \tag{35}$$

where L_R is the number of millions of revolutions of the sun gear at which the probability of non-destruction of the epicycle is equal to S_R.

6 Reliability and load-bearing capacity of satellites

The final set of planetary drive components that have finite fatigue life are the planetary gears themselves. These gears engage with both the sun gear and the epicycle. However, as it can be seen from figures 2 and 3, the load in two engagement zones is perceived by opposite profiles of the satellite teeth. Thus, even if the satellites are not double-crowned $R_{PS} = R_{PR}$, the growth of fatigue chipping sites in each engagement zone occurs independently until an increased dynamic load appears. In the model under consideration, the assumption is made that the indicated increased dynamic load occurs after the onset of destruction, so we will assume that both sources of destruction develop independently of each other.

The number of loading cycles that each satellite perceives, depending on the number of revolutions of the sun gear, is calculated using the formulas in the table as the number of revolutions of the satellite relative to the carrier:

$$l_P = \frac{R_R R_S L_R}{R_R R_{PS} + R_S R_{PR}}. \tag{36}$$

This number of loading cycles is the same for the zone of engagement with the sun gear and the epicycle, although the nature of the fatigue fracture in these zones is different.

The probability of non-destruction of the satellite S_P is determined as the product of the probabilities for each engagement zone

$$S_P = S_{PS}^{N_{PS}} \cdot S_{PR}^{N_{PR}} \tag{37}$$

Solving equation (19) for each engagement zone together with equations (36) and (37), we obtain an expression for the reliability of the satellite

$$\log\frac{1}{S_P} = N_{PS}\log\frac{1}{0,9}\left\{\frac{R_R R_S L_P}{(R_S R_{PR} + R_R R_{PS})\, l_{PS10}}\right\}^{e_G} + N_{PR}\log\frac{1}{0,9}\left\{\frac{R_R R_S L_P}{(R_S R_{PR} + R_R R_{PS})\, l_{PR10}}\right\}^{e_G} \tag{38}$$

Durability based on the 90% probability of non-destruction of the satellite, at a 90% probability condition of non-destruction of its teeth, is obtained using the formula

$$L_{P10} = \left[\frac{R_S R_{PR} + R_R R_{PS}}{R_R R_S}\right] \cdot \left\{\frac{l_{PS10} l_{PR10}}{[N_{PS}(l_{PR10})^{e_G} + N_{PR}(l_{PS10})^{e_G}]^{\frac{1}{e_G}}}\right\} \tag{39}$$

The initial dynamic load-bearing capacity D_P of the satellite is the magnitude of the torque on the shaft connected to the sun gear, at which the probability of non-destruction of the satellite is 90% for 106 revolutions of the sun gear:

$$D_P = \left[\frac{R_S R_{PR} + R_R R_{PS}}{R_R R_S}\right]^{\frac{1}{e_G}} \cdot \left\{\frac{n R_S R_{PR} C_S C_R}{\left[N_{PS}(R_{PS} C_R)^{P_G e_G} + N_{PR}(R_{PS} C_S)^{P_G e_G}\right]^{\frac{1}{P_G e_G}}}\right\} \tag{40}$$

The formula for calculating the durability of the satellites, expressed in terms of the torque on the shaft connected to the sun gear, provided that 90% of the satellites do not collapse, has the form

$$L_{P10} = \left(\frac{D_P}{T_i}\right)^{P_G}. \tag{41}$$

So, the initial quantitative characteristics necessary to calculate the reliability and durability distribution curve for all drive gears have been determined. Finally, the equation of the satellite reliability probability distribution curve is as follows:

$$\log\frac{1}{S_P} = \log\frac{1}{0,9} \cdot \left(\frac{L_P}{L_{P10}}\right)^{e_G}. \tag{42}$$

7 Reliability and load-bearing capacity of the transmission as a whole

The multiplier rule can be used to calculate the probability of non-destruction of the transmission as a whole, which includes satellite bearings, the sun gears, satellites and the epicycles:

$$S_T = S_B^n \cdot S_S \cdot S_P^n \cdot S_R. \tag{43}$$

The probability distribution curve of non-destruction of the gear as a whole can be obtained by substituting equations (18), (28), (35) and (42) into the expression for the natural logarithm of the equation inverse to equation (43)

$$\log\frac{1}{S_T} = \log\frac{1}{0,9} \cdot \left\{n\left(\frac{L_T}{L_{B10}}\right)^{e_W} + \left(\frac{L_T}{L_{S10}}\right)^{e_G} + n\left(\frac{L_T}{L_{P10}}\right)^{e_G} + \left(\frac{L_T}{L_{R10}}\right)^{e_G}\right\} \tag{44}$$

Since the durability of all drive elements is determined in the same units (in revolutions of the sun), we denote them by the symbol L_T included in the formula for calculating the probability of non-destruction S_T of the drive as a whole.

Unfortunately, equation (44) does not strictly describe the Weibull relationship between system durability and reliability. This equation will strictly correspond to the true Weibull distribution only in the case of $e_W = e_G$, which does not happen in the general case. In figure 4 shows the curve corresponding to equation (44) in Weibull coordinates.

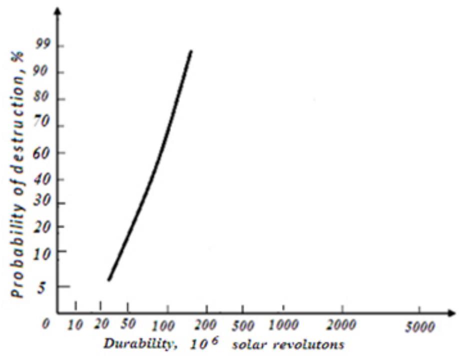

Figure 4: Weibull curve characterizing the dependence S_T from L_T for planetary gear as a whole

This and subsequent figures show the curves obtained for the planetary transmission shown in figure1, in which the sun gear had $z = 20$, epicycle $z = 85$, three satellites were $z = 40$ at the point of engagement with the sun gear and $z = 25$ at the place of engagement with the epicycle. The transmission module is 1,59 mm (diametric pitch $P_d = 16$), the top of the crown is 9,19 mm. The gears had a standard involute profile with a profile angle of 200 and were made of AISI 9310 steel. The torque on the transmission input shaft associated with the sun gear is *48 Nm.* In the examples discussed in this work, the Weibull curve exponent for bearings was 1.2, and for gears - 2.5. For this balanced version, the initial dynamic load-bearing capacity of each satellite bearing is *15250 N.*

This curve can be approximated by a straight line using the least squares method in the range $0,5 \leq S_T \leq 0,95$. The slope angle of the approximating straight line is called the slope angle of the Weibull system e_T, and the durability of the system characterized by a straight line at $S_T = 0,9$ is called the durability of the system with 90% probability.

The exact durability L_{T10} can be calculated by substituting $S_T = 0,9$ into equation (44) and solving by successive approximations with respect to L_{T10}:

$$1 = n\left(\frac{L_{T10}}{L_{B10}}\right)^{e_B} + \left(\frac{L_{T10}}{L_{S10}}\right)^{e_G} + n\left(\frac{L_{T10}}{L_{P10}}\right)^{e_G} + \left(\frac{L_{T10}}{L_{R10}}\right)^{e_G}. \tag{45}$$

For the cases studied in this work, the Weibull L_{T10} durability differed from the durability L_{T10} calculated using formula (45) by no more than 1%. Since this error is significantly less than the difference between the values obtained as a result of tests and the Weibull durability of the elements, such an approximation seems justified. If one drive element is weaker than the others, the overall drive reliability model and least squares fit approximates the Weibull model for the weaker element. This is illustrated in figure 5, which shows several Weibull curves for the satellite bearing, the sun gear, the drive as a whole, and $L_{B10} = 1,7 \cdot 10^6$ cycles, $L_{S10} = 84,3 \cdot 10^6$ cycles, and $L_{T10} = 0,68 \cdot 10^6$ cycles. This example differs from the example in figure 4 in that the satellite bearings are less durable and the dynamic load-bearing capacity is equal to *3460 N.* In this case, the life of the bearing determines the life of the drive, and the Weibull curve for the drive is 1.2.

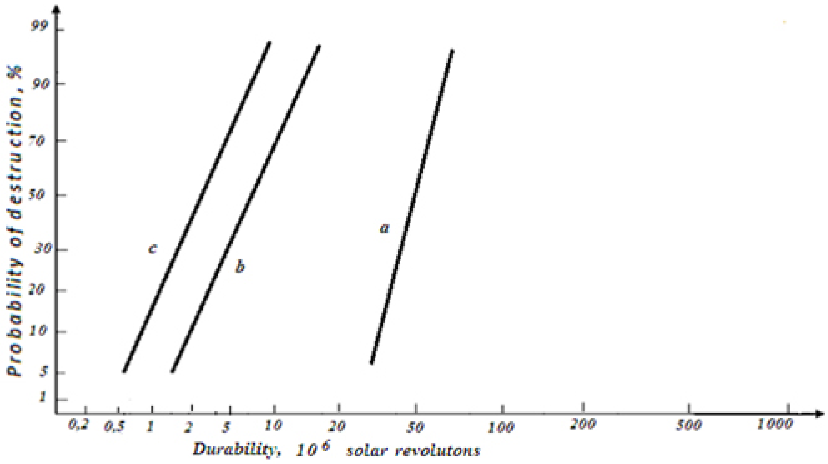

Figure 5: Weibull curves for the sun gear (a), satellite bearings (b) and Gear as a whole (with) with weakened satellite bearings

If the load-bearing capacity of the bearing approaches the load-bearing capacity of the sun gear teeth, then the actual drive reliability curve is very different from the Weibull curve approximated by the least squares method. This is illustrated in figure 6, which shows a set of Weibull curves for the satellite bearing, the sun gear and the drive as a whole with the following parameters of the latter: $L_{B10} = 207 \cdot 10^6$ sun gear revolutions, $L_{S10} = 84,3 \cdot 10^6$ sun gear revolutions and $L_{T10} = 55,5 \cdot 10^6$ sun gear revolutions (the same case is shown in figure 4). Here the Weibull exponent is 1.84.

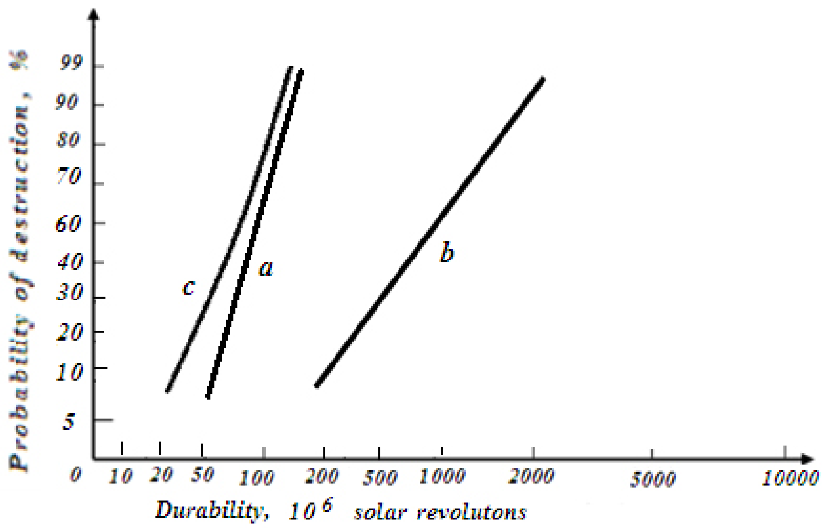

Figure 6: Weibull curves for the sun gear (a), satellite bearings (b) and gear as a whole (with) the strength of the elements of which is the same

For this straight-line Weibull curve for the drive, the design formula is

$$\log \frac{1}{S_T} = \log \frac{1}{0,9} \left(\frac{L_T}{L_{T10}} \right)^{e_T}. \tag{46}$$

The initial dynamic load-bearing capacity D_T is equal to the value of the torque on the input shaft associated with the sun gear, with 90% reliability and durability of $L_{T10} = 10^6$ the sun gear rotations. Substituting $S_T = 0,9$ into equation (44) and substituting equations (17), (27),

(34) and (41) we obtain $L_{T10} = 1$:

$$1 = n\left[\frac{D_T}{D_B}\right]^{P_B e_B} + \left[\frac{D_T}{D_S}\right]^{P_G e_G} + n\left[\frac{D_T}{D_P}\right]^{P_G e_G} + \left[\frac{D_T}{D_R}\right]^{P_G e_G}. \quad (47)$$

The initial dynamic load-bearing capacity of the drive can be obtained by calculating this formula using the method of successive approximations, since the indicators of the degree and magnitude of the load-bearing capacity of the transmission elements are known. It can also be obtained from equation (45) by determining a number of values L_{T10} corresponding to a number of torque values on the input shaft T_i and plotting the dependence T_i from L_{T10} in natural logarithmic coordinate system. The value T_i corresponding $L_{T10} = 10^6$ revolutions of the sun is equal to the initial dynamic load-bearing capacity of the drive. Figure 7 shows the dependence T_i on L_{T10} in a natural logarithmic coordinate system for a drive characterized by the data from figure 6. The exponent of P_T the load-durability relationship is negative. In the examples discussed in this work, $P_B = 3,3$, and $P_G = 4,3$. In this example, the dynamic load-bearing capacities are: $D_B = 242\,H \cdot m,\ D_S = 135\,H \cdot m,\ D_T = 134\,H \cdot m$. As for the Weibull model of the entire drive, an approximate load-life relationship is obtained by least squares method for a certain range of input torques (i.e. $0,1D_T \leq T_i \leq D_T$). Taking into account the indicated approximation, the dependence of the load on the durability of the system is determined by the equation

$$L_{T10} = \left(\frac{D_T}{T_i}\right)^{P_T}. \quad (48)$$

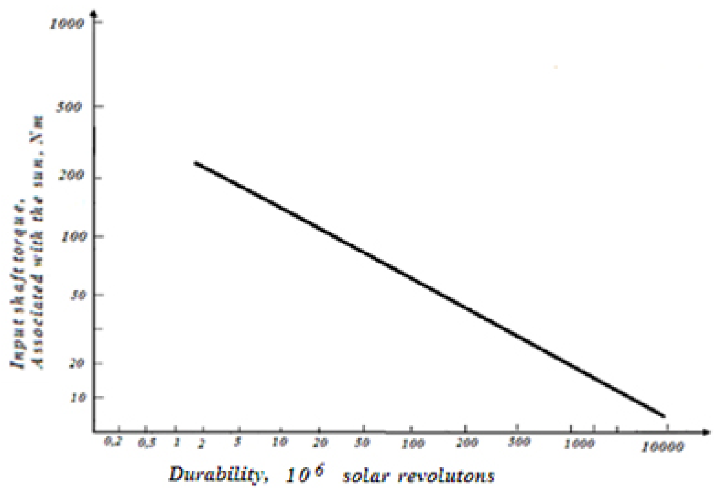

Figure 7: Dependence of load on durability for planetary gear as a whole

In the case illustrated in figure 7, the exponent of the load-durability curve P_T is 3.8. As for the Weibull model, the dynamic load-bearing capacity of the drive determines the weakest link, and the exponent of the load-durability curve and the load-bearing capacity of the entire system approach the values of the specified parameters of the weakest link.

8 Conclusion

A model of the reliability of a planetary transmission has been developed for use in the probabilistic calculation of drives of this type. The transmission scheme is as follows: the epicycle is stationary, the input shaft is connected to the sun gear, and the satellite carrier is connected to the output shaft. It is assumed that the input and output shafts are coaxial and collinear to the torque vectors; lateral loads are not taken into account.

The drive reliability model is based on bearing and gear reliability models, which in turn are two-dimensional Weibull curves of the probability distribution of reliability as a function of durability. The initial dynamic load-bearing capacity of the drive and its durability, calculated based on a 90% probability of non-destruction, are expressed in terms of the number of millions of revolutions of the input shaft associated with the sun gear. The specified durability and load-bearing capacity are determined as exact functions of the lives and load-bearing capacities of the drive elements. However, due to the fact that the probability distributions of reliability for bearings and gears are different, the Weibull model for planetary gears is an approximate one. This model is characterized by durability due to a 90% probability of non-destruction, an indicator of the degree of the load-durability curve.

The following results are obtained.

1. A reliability model of a spur cylindrical planetary gear has been developed, including satellite bearings; the possibility of using double-crown satellites is provided.

2. It was discovered that Weibull reliability distribution curves with different indicators do not obey the law of mathematical closure.

3. Indicators for the straight-line Weibull relationship and the load-durability relationship were obtained for the Weibull model of a certain system consisting of various elements.

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